Binomial Expansion – Coefficients

Exercise 8D

1 Find the coefficient of x^3 in the binomial expansion of:

a $(3 + x)^5$ **b** $(1 + 2x)^5$ **c** $(1 - x)^6$ **d** $(3x + 2)^5$

e $(1 + x)^{10}$ **f** $(3 - 2x)^6$ **g** $(1 + x)^{20}$ **h** $(4 - 3x)^7$

i $\left(1 - \frac{1}{2}x\right)^6$ **j** $\left(3 + \frac{1}{2}x\right)^7$ **k** $\left(2 - \frac{1}{2}x\right)^8$ **l** $\left(5 + \frac{1}{4}x\right)^5$

(P) **2** The coefficient of x^2 in the expansion of $(2 + ax)^6$ is 60. Find two possible values of the constant a.

Problem-solving

$a = 2$, $b = ax$, $n = 6$. Use brackets when you substitute ax.

(P) **3** The coefficient of x^3 in the expansion of $(3 + bx)^5$ is -720. Find the value of the constant b.

(P) **4** The coefficient of x^3 in the expansion of $(2 + x)(3 - ax)^4$ is 30. Find the three possible values of the constant a.

(E/P) **5** When $(1 - 2x)^p$ is expanded, the coefficient of x^2 is 40. Given that $p > 0$, use this information to find:

a the value of the constant p **(6 marks)**

b the coefficient of x **(1 mark)**

c the coefficient of x^3 **(2 marks)**

Problem-solving

You will need to use the definition of $\binom{n}{r}$ to find an expression for $\binom{p}{2}$.

(E/P) **6** **a** Find the first three terms, in ascending powers of x, of the binomial expansion of $(5 + px)^{30}$, where p is a non-zero constant. **(2 marks)**

b Given that in this expansion the coefficient of x^2 is 29 times the coefficient of x work out the value of p. **(4 marks)**

E/P **7** **a** Find the first four terms, in ascending powers of x, of the binomial expansion of $(1 + qx)^{10}$, where q is a non-zero constant. **(2 marks)**

b Given that in the expansion of $(1 + qx)^{10}$ the coefficient of x^3 is 108 times the coefficient of x, work out the value of q. **(4 marks)**

E/P **8** **a** Find the first three terms, in ascending powers of x of the binomial expansion of $(1 + px)^{11}$, where p is a constant. **(2 marks)**

b The first 3 terms in the same expansion are 1, $77x$ and qx^2, where q is a constant. Find the value of p and the value of q. **(4 marks)**

E/P **9** **a** Write down the first three terms, in ascending powers of x, of the binomial expansion of $(1 + px)^{15}$, where p is a non-zero constant. **(2 marks)**

b Given that, in the expansion of $(1 + px)^{15}$, the coefficient of x is $(-q)$ and the coefficient of x^2 is $5q$, find the value of p and the value of q. **(4 marks)**

E/P **10** In the binomial expansion of $(1 + x)^{30}$, the coefficients of x^9 and x^{10} are p and q respectively. Find the value of $\frac{q}{p}$. **(4 marks)**

Challenge

Find the coefficient of x^4 in the binomial expansion of: **a** $(3 - 2x^2)^9$ **b** $\left(\frac{5}{x} + x^2\right)^8$